

# Determination of the temperature distribution in an extended surface with a non-uniform heat transfer coefficient

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**Abstract**—Assuming that the heat transfer coefficient is a power function of the difference between the temperature of a straight fin of rectangular profile and that of the fluid surrounding the fin, the values of the exponent in the power function are given for which a closed-form solution for one-dimensional temperature distribution in the fin can be derived practically. For nucleate boiling of many significant liquids, this closed-form solution is presented in detail together with the fin effectiveness and fin efficiency.

## INTRODUCTION

FOR STEADY-STATE, *one-dimensional* mathematical analysis of an extended surface, a uniform heat transfer coefficient was assumed in practically all the studies presented in the relative literature. This, together with the additional appropriate assumptions, yielded an explicit closed-form solution for the temperature distribution in an extended surface [1–3]. In a few of these studies, this coefficient was assumed to be a function of space coordinate which also resulted in a similar type solution to that previously quoted [4–6].

The closed-form explicit formulae for the prediction of the temperature distributions in the most used extended surfaces are given in a well-known paper [2] and book [3].

If the heat transfer coefficient is non-uniform and heat conduction is either one- or multi-dimensional, the temperature distribution in an extended surface is determined by a numerical method which also eliminates some assumptions necessary to obtain a closed-form solution [7–9]. As quoted from a recent paper [10], “for the design engineer, the desirability of simple closed form expressions may well outweigh considerations of rigor and exactness”.

For many practical applications (for natural convection and nucleate boiling for example), the heat transfer coefficient does not obey Newton’s cooling law. In the case of natural convection, it is a function of the temperature difference between a heat exchanging surface and a fluid surrounding this surface. For nucleate pool boiling or nucleate flow boiling with low steam qualities, it is a function of the wall superheat, i.e. the difference between the temperature of a heated surface and the saturation temperature of a liquid boiling on this surface. For these types of boiling conditions, the heat transfer coefficient is expressed as follows:

$$h = a_1 \theta^n \quad (1)$$

where  $a_1$  and  $n$  are constants for a given boiling surface, liquid and pressure. For engineering calculations and for water, refrigerants and some organic liquids  $n$ , which is a non-dimensional constant, is equal to about 2 [11]. If all types of boiling conditions are taken into account,  $n$  may vary between  $-6.6$  and  $5$  [8].

The object of this paper is to investigate whether a closed-form expression exists for a one-dimensional temperature distribution in a straight fin of rectangular profile for each value of  $n$  in equation (1). The case when  $n > 0$  is dealt with in detail.

## DIFFERENTIAL EQUATION OF TEMPERATURE DISTRIBUTION

A thin, straight fin of rectangular profile, as shown in Fig. 1, is now being considered. It is the most simple form of fin from the aspect of mathematical analysis. It is assumed that there is one-dimensional steady-state heat conduction through the fin, a constant thermal

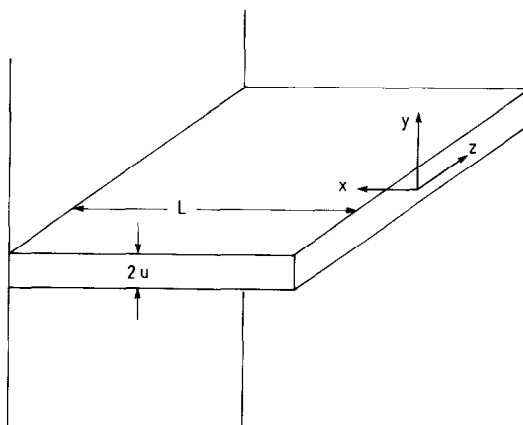


FIG. 1. Straight fin of rectangular profile.

## NOMENCLATURE

$a_1, \dots, a_{11}$	dimensional constants	$q$	heat flux [ $\text{W m}^{-2}$ ]
$b_1, b_2$	non-dimensional constants	$r, t$	non-dimensional temperature difference or wall superheat
$C$	constant of integration	$u$	half fin thickness [m]
$F(\phi/\alpha)$	Legendre's normal elliptic integral of the first kind	$x, y, z$	Cartesian coordinates [m].
$f$	fin effectiveness	Greek symbols	
$h$	heat transfer coefficient [ $\text{W m}^{-2} \text{K}^{-1}$ ]		
$i$	an index	$\alpha$	modular angle [rad]
$J$	a function of $\phi$	$\theta$	temperature difference between fin and surrounding fluid or wall superheat at point $x$ [K]
$k$	thermal conductivity of fin material [ $\text{W m}^{-1} \text{K}^{-1}$ ]	$\phi$	amplitude [rad].
$L$	fin length [m]	Subscripts	
$m, n$	dimensionless constants		
$p$	first derivative of $\theta$ [ $\text{K m}^{-1}$ ]	$b$	refers to fin base (i.e. $x = L$ )
$Q$	rate of heat flow through entire fin per unit length in $z$ -direction [ $\text{W m}^{-1}$ ]	$c$	refers to fin tip (i.e. $x = 0$ ).

conductivity for fin material, no heat sources in the fin itself, a uniform temperature at the fin base, a uniform temperature for the surrounding fluid, negligible heat transfer from the fin tip and a constant fin thickness. With the exception of the last assumption, all the others were used in most of the analytic studies presented in the literature [1–6].

If  $hu/k$ , the Biot number for the fin, is less than 0.1, then the effect on the rate of heat flow from the fin of heat conduction in  $y$ -direction appears to be quite negligible (i.e. less than 1%) [12].

For the assumptions made, the differential equation of the temperature distribution in the fin becomes [2, 3]

$$\frac{d^2\theta}{dx^2} = \frac{h}{ku}\theta. \quad (2)$$

The boundary conditions are:

$$\theta = \theta_b \quad \text{for } x = L, \quad (3)$$

and

$$\left(\frac{d\theta}{dx}\right) = 0 \quad \text{for } x = 0. \quad (4)$$

The total rate of heat flow per unit length leaving the fin in the  $z$ -direction is:

$$Q = 2 \int_0^L h\theta \, dx. \quad (5)$$

If assumptions similar to those quoted above are considered, equation (2) then also applies to a pin fin (or a rod protruding from a heat source) if  $u$  is replaced by the ratio of the area of the cross-section of the pin to its circumference and if  $L$  is the pin length [3]. The diameter of the pin is constant and the total rate of heat flow through the pin is given by equation (5) if 2 is replaced by the circumference of the pin.

#### Solution of the differential equation

The heat transfer coefficient  $h$  in equation (2) is expressed in equation (1). For  $n \neq 0$ , equation (2) is a highly non-linear differential equation. In order to solve it, a new variable  $p = d\theta/dx$ , is introduced. The derivative of this variable with respect to  $x$  is:

$$\frac{dp}{d\theta} \frac{d\theta}{dx} = \frac{d^2\theta}{dx^2}. \quad (6)$$

After inserting  $h$  given in equation (1) into equation (2), the subtraction of the latter from equation (6) results in:

$$\frac{dp}{d\theta} p = a_2 \theta^{n+1} \quad (7)$$

where

$$a_2 = a_1/(ku). \quad (8)$$

If  $n \neq -2$ , the integration and rearrangement of equation (7) gives:

$$p = \frac{d\theta}{dx} = \left( 2a_2 \frac{\theta^{n+2}}{n+2} + 2C \right)^{0.5}. \quad (9)$$

The use of the second boundary condition [i.e. equation (4)] yields the value of the constant of integration

$$C = -a_2 \theta_e^{n+2}/(n+2). \quad (10)$$

By now non-dimensionalizing the temperature difference (or wall superheat)  $\theta$  as

$$t = \theta/\theta_e \quad \text{or } r = (\theta/\theta_e)^{n+2} \quad (11)$$

and inserting the value of the constant of integration in equation (9), this equation is transformed into:

$$\frac{dt}{\{a_3(t^{n+2}-1)\}^{0.5}} = dx \quad (12)$$

or

$$\frac{r^{-(n+1)/(n+2)} dr}{(n+2) \{a_3(r-1)\}^{0.5}} = dx \quad (13)$$

where

$$a_3 = 2a_2\theta_e^n/(n+2). \quad (14)$$

In order to calculate the temperature difference or wall superheat  $\theta$ , equation (13) [or equation (12)] should be integrated first; thereafter the value of  $\theta_e$  can be determined from the integrated equation using the first boundary condition [i.e. equation (3)]. The integration of the RHS of equation (13) is straightforward. The values of  $n$  for which the LHS of this equation can be integrated analytically will now be briefly discussed.

Let  $b_1$  and  $b_2$  be defined as follows:

$$b_1 = 1/(n+2) \quad (15)$$

$$b_2 = -n/\{2(n+2)\}. \quad (16)$$

If either  $b_1$  or  $b_2$  is a positive or negative integer or zero, the integration of the LHS of equation (13) can be reduced to that of a rational function [13] with the aid of the formulae given in a mathematical textbook [13] or handbook [14, 15]. The latter is carried out using ordinary (i.e. algebraic, logarithmic and circular) functions [13].

If neither  $b_1$  nor  $b_2$  is an integer or zero, then the integral of the LHS of equation (13) cannot be expressed by ordinary functions in accordance with a theorem proved by Tchebichev [16]. In this case, the integration of the LHS of this equation can be reduced to that of an equation in which the powers of  $r$  and  $[a_3(r-1)]$  are between two successive integers; say between 0 and 1. Since the latter is susceptible to further reduction, it should be considered as a new transcendental function [13]. The determination of this integration is practically improbable with analytic methods as quoted from ref. [13]. This implies that no closed-form expression exists for the solution of equation (2). For two values of  $n$  however, the result of this integration is given as an analytic function in the literature as will be discussed below.

#### Positive values of $n$

This case is typical for natural convection and nucleate boiling. It follows from equations (15) and (16) that neither  $b_1$  nor  $b_2$  is an integer or zero if  $n$  is a positive real number. This means that the integration of the LHS of equation (13) is practically impossible with analytic methods. However, for  $n = 1$  and  $n = 2$  the result of this integration is known as an analytic function. For these two integers, the LHS of equation (12), which is identical to that of equation (13), is the integrand of a normal elliptic integral of the first kind. This elliptic integral can be transformed to Legendre's (incomplete) normal integral of the first kind, whose value is known [14, 15, 17].

As stated before, for nucleate pool boiling and

nucleate flow boiling with low steam qualities,  $n$  can be taken as being equal to 2 for many liquids. For this value of  $n$ , the integration of the LHS of equation (12) from  $t = 1$  to  $t = t$  and that of its RHS from  $x = 0$  to  $x = x$ , yields the non-dimensional wall superheat (or temperature difference)  $t$  as [17]

$$mF(\phi/\alpha) = a_3^{0.5}x \quad (17)$$

where

$$m = 2^{-0.5} \quad (18)$$

$$\cos \phi = 1/t = \theta_e/\theta \quad \text{for } 0 \leq \phi \leq \pi \quad (19)$$

$$\alpha = \pi/4 \quad (20)$$

$F(\phi/\alpha)$  in equation (17), Legendre's (incomplete) normal elliptic integral of the first kind, is tabulated in ref. [17] as a function of  $\phi$  and  $\alpha$ . This integral is also given as an analytical function in ref. [15], i.e.

$$F(\phi/\alpha) = \sum_{i=0}^{\infty} \left( \frac{-1}{i} \right) (-\sin^2 \alpha)^i J_{2i} \quad (21)$$

where

$$J_{2i} = \frac{2i-1}{2i} J_{2i-2} - \frac{1}{2i} \sin^{2i-1} \phi \cos \phi \quad (22)$$

$$J_0 = \phi \quad (23)$$

$F(\phi/\alpha)$  is valid for  $0 \leq \phi \leq \pi$ .

In order to utilize equation (17),  $\theta_e$ , the wall superheat (or the temperature difference) at the fin tip should be known. This can be determined as follows: the application of the first boundary condition [i.e. equation (3)] to equations (17) and (19) yields:

$$mF(\phi_b/\alpha) = a_3^{0.5}L \quad (24)$$

$$\cos \phi_b = \theta_e/\theta_b. \quad (25)$$

In these equations, only the values of  $\phi_b$  and  $\theta_e$  are unknown. In order to calculate these values, a value for  $\theta_e$  is assumed;  $\phi_b$  is determined from equation (25) and  $F(\phi_b/\alpha)$  from equation (24) and equation (21) [or from the tabulated values of  $F(\phi/\alpha)$ ]. The value of  $\theta_e$  is iterated until the calculated two  $F(\phi_b/\alpha)$  values are identical.

In order to calculate  $\theta$  for a given value of  $x$ ,  $F(\phi/\alpha)$  is first calculated with equation (17). The value of  $\phi$  corresponding to this  $F(\phi/\alpha)$  is then determined with an iteration procedure from equation (21) or from the tabulated values of  $F(\phi/\alpha)$ . Using the value of this  $\phi$ ,  $\theta$  is calculated using equation (19).

For the determination of  $x$  for a given  $\theta$  (i.e.  $\theta_e < \theta < \theta_b$ ),  $\phi$  is first solved from equation (19), thereafter  $F(\phi/\alpha)$  is solved from equation (21) [or from the tabulated values of  $F(\phi/\alpha)$ ] and finally  $x$  is solved from equation (17). The foregoing method eliminates the preceding iteration procedure.

For  $n = 2$  and pin fins, Petukhov *et al.* [18] gave the solution of equation (2) as the difference of two elliptic integrals of the first kind, the boundary conditions used being different from those of present study.

For  $n = 1$ , the temperature distribution in the fin is again given by equation (17) [17], where:

$$m = 3^{-0.25} \tag{26}$$

$$\cos \phi = \frac{\sqrt{3+1-\theta/\theta_e}}{\sqrt{3-1+\theta/\theta_e}} \text{ for } 0 \leq \phi \leq \pi \tag{27}$$

$$\alpha = \pi/12. \tag{28}$$

In order to calculate  $\theta_e, \theta$  for a given  $x$  and  $x$  for a given  $\theta$ , the procedures explained for the case in which  $n = 2$  are used, taking into consideration the fact that equation (27) replaces (19).

$F(\phi/\alpha)$  is usually tabulated for  $0 \leq \phi \leq \pi/2$ , while equation (21) is valid for  $0 \leq \phi \leq \pi$ . This equation converges gradually for  $n = 2$  (i.e.  $i \cong 10$ ) and rapidly for  $n = 1$  (i.e.  $i = 2$ ). The tabulated values of  $F(\phi/\alpha)$  can be used with the following formula if  $\phi > \pi/2$ :

$$F(\phi/\alpha) = 2F[(\pi/2)/\alpha] - F[(\pi - \phi)/\alpha]. \tag{29}$$

The derivation of this equation is straightforward from the definition of Legendre's normal elliptic integral of the first kind and trigonometry [14, 15, 17].

In accordance with the boundary condition given by equation (4), no heat at all is lost at the tip of the fin. As suggested by Jakob [3], and for constant heat transfer coefficient, the small heat loss that will actually occur at the fin tip can be taken into account approximately by using the fin length as  $(L + u)$  instead of  $L$ . For  $n = 1$  and 2, such a correction appears to be of secondary importance considering that  $Q$  is a power function with a positive exponent.

For  $n = 0$ , the heat transfer coefficient is constant. For this case,  $b_2$  becomes zero and equation (13) can be integrated using ordinary functions, yielding:

$$\theta = \theta_b \frac{\cosh(a_2^{0.5} x)}{\cosh(a_2^{0.5} L)}. \tag{30}$$

As demonstrated above, a closed-form solution for the temperature distribution in a straight fin of rectangular profile exists for  $n = 1$  and 2 if  $n > 0$ . However, this solution is not an explicit function of the independent variable  $x$ , contrary to the solution obtained for a constant heat transfer coefficient [i.e. see equation (30)]. Once again however, this does not seem to be a serious objection for a practising engineer since, in most cases, he needs the fin efficiency or fin effectiveness and the rate of heat flow through the fin. These can be calculated with simple formulae if  $\theta_e$  is known, as can be deduced from equations (5), (31) and (34).

Negative values of n

This case is typical for transition (partial) film boiling and condensation. The values of  $n$  for which the LHS of equation (13) can be integrated with the aid of ordinary functions can be obtained from equations (15) and (16) using Tchebichev's theorem. These values are given in Table 1 if  $n$  is a negative integer or a negative number with one digit. For other values of  $n$ , including  $n = -2$ ,

Table 1. Negative values of  $n$ , for which equation (13) can be practically integrated

$n$	$n$
0	-2.1
-1.0	-2.2
-1.5	-2.4
-1.6	-2.5
-1.8	-3.0
-1.9	-4.0

the use of a numerical method seems almost inevitable, at least, for practical applications.

FIN EFFECTIVENESS AND FIN EFFICIENCY

These are useful criteria to characterize an extended surface. The fin effectiveness is the ratio of  $Q$ , the heat transferred through the base of a fin, to that which would be transferred through the same base area if the fin were not there, the base temperature remaining constant [2]. Making use of equations (1) and (5), the fin effectiveness obtained is thus:

$$f = \frac{2 \int_0^L h \theta \, dx}{2u\theta_b h_b} = \frac{\int_0^L \theta^{n+1} \, dx}{u\theta_b^{n+1}}. \tag{31}$$

In order to determine the integral in the numerator of equation (31), it is first written as a function of the dimensionless temperature difference  $r$ , using equations (11) and (13), i.e.

$$f = \frac{\theta_e^{n+1} \int_1^{r_b} \{a_3(r-1)\}^{-0.5} \, dr}{(n+2)u\theta_b^{n+1}} \tag{32}$$

where

$$r_b = (\theta_b/\theta_e)^{n+2}. \tag{33}$$

The determination of the integral in equation (32) is straightforward [14, 15], yielding

$$f = \frac{2\theta_e^{n+1} \left\{ a_3 \left[ \left( \frac{\theta_b}{\theta_e} \right)^{n+2} - 1 \right] \right\}^{0.5}}{ua_3(n+2)\theta_b^{n+1}}. \tag{34}$$

Equation (34) is valid for all real values of  $n$  with the exception of  $n = -2$ .

For a non-uniform heat transfer coefficient, the fin effectiveness seems a more proper criterion than the fin efficiency which is the ratio of the total rate of heat flow through the entire fin to the rate of heat flow calculated using the temperature difference and heat transfer coefficient at the fin base and the total heat dissipation area of the fin. The fin efficiency is given by equation (34) if the RHS of equation (34) is multiplied by  $(u/L)$ , as can be deduced from the definition of the fin effectiveness and fin efficiency.

The practical significance of equation (34) is obvious. If  $\theta_b$  (i.e. the boundary value) and  $\theta_e$  are known, it gives the fin effectiveness or fin efficiency without using any other temperature difference in the fin. This means that the one-dimensional analysis of the fin with a numerical method can be substantially shortened if  $h = a_1\theta^n$ . Furthermore note that  $Q = 2u\theta_b h_b f$  [see equation (31)].

#### FURTHER COMMENTS ON NON-UNIFORM HEAT TRANSFER COEFFICIENT

There are many practical situations for which the heat flux can be given by equation (35)

$$q = a_1\theta^3 + a_4\theta^2 + a_5\theta + a_6 \quad (35)$$

where  $a_4$ – $a_6$  are constants for a given geometry, pressure and temperature. For subcooled nucleate flow boiling, for example, Rohsenow's superposition rule [19, 20] yields equation (35) if  $a_4$  in it is equal to zero.  $\theta$  is the wall superheat. For the same type of boiling, the author [21] also proposed this equation in the case where  $a_4$  and  $a_6$  in it are equal to zero. Note that equation (35) applies also to a fin if heat is generated uniformly in it, and if the heat transfer coefficient is given by polynomials of second or the first degree.

After replacing  $h\theta$  (i.e. heat flux) in equation (2) by the heat flux given by equation (35), equation (2) can be reduced to [see equations (7) and (9)]:

$$\int_{\theta_e}^{\theta} \frac{d\theta}{(a_7\theta^4 + a_8\theta^3 + a_9\theta^2 + a_{10}\theta + a_{11})^{0.5}} = \int_0^x dx \quad (36)$$

where  $a_7$ – $a_{11}$  are constants.  $a_{11}$ , which is a function of  $\theta_e$ , is calculated with the second boundary condition. The determination of equation (36) can be carried out by reducing its LHS to Legendre's normal elliptic integral of the first kind, yielding [14, 15]

$$mF(\phi/\alpha) = x \quad (37)$$

and

$$\phi = \phi(t). \quad (38)$$

The formulae for the derivation of  $m$ ,  $\phi$  and  $\alpha$  are given in refs. [14, 15]. These formulae differ in accordance with the type of the zeros of the polynomials equation in the denominator of equation (36). In order to save space, they have been omitted here. After the determination of  $m$ ,  $\phi$  and  $\alpha$ ,  $\theta_e$  is obtained with a procedure identical to that explained before. If  $a_1$  and  $a_4$  in equation (35) are equal to zero, then equation (36) is calculated with ordinary functions.

#### CONCLUSIONS

If the heat transfer coefficient is a power function of the temperature difference between a straight fin of rectangular profile and the fluid surrounding the fin or of the wall superheat, a closed-form solution for one-

dimensional temperature distribution in the fin can be derived for a limited number of cases.

When this coefficient is an increasing power function of the temperature difference or wall superheat, the foregoing closed-form solution appears to be practically available for two values of the exponent in the power function, i.e.  $n = 1$  and 2. These solutions are presented in detail. The latter exponent applies to a significant type of heat transfer mode for many liquids, i.e. nucleate pool boiling and nucleate flow boiling with low steam qualities.

A formula is given to determine the fin effectiveness and fin efficiency, which can substantially shorten the one-dimensional analysis of a fin with a numerical method if  $h \approx \theta^n$ .

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#### DETERMINATION DE LA DISTRIBUTION DE TEMPERATURE SUR UNE SURFACE ETENDUE AVEC UN COEFFICIENT DE TRANSFERT THERMIQUE NON UNIFORME

**Résumé**—Admettant que le coefficient de transfert thermique est une fonction puissance de la différence de température entre une ailette plane de profil rectangulaire et le fluide environnant, et que les valeurs de l'exposant de la fonction puissance sont données, on obtient pratiquement la solution de la distribution unidimensionnelle de la température de l'aillette. Pour l'ébullition nucléée de plusieurs liquides significatifs, cette solution analytique est présentée en détail avec le rendement et l'efficacité de l'aillette.

#### BESTIMMUNG DER TEMPERATURVERTEILUNG AN EINER RIPPEN-OBERFLÄCHE MIT NICHT EINHEITLICHEN WÄRMEÜBERGANGSKOEFFIZIENTEN

**Zusammenfassung**—Unter der Voraussetzung, daß der Wärmeübergangskoeffizient durch eine Potenzfunktion der Temperaturdifferenz zwischen einer geraden Rechteck-Rippe und dem umgebenden Fluid dargestellt werden kann, werden die Werte der Exponenten dieser Potenzfunktion angegeben. Hiermit kann eine Lösung der eindimensionalen Temperaturverteilung in der Rippe in geschlossener Form einfach formuliert werden. Für das Blasensieden von vielen wichtigen Flüssigkeiten wird diese geschlossene Lösung im einzelnen in Verbindung mit dem Rippenwirkungsgrad vorgestellt.

#### ОПРЕДЕЛЕНИЕ РАСПРЕДЕЛЕНИЯ ТЕМПЕРАТУРЫ НА ВЫТЯНУТОЙ ПОВЕРХНОСТИ С НЕОДНОРОДНЫМ КОЭФФИЦИЕНТОМ ТЕПЛООБМЕНА

**Аннотация**—В предположении, что коэффициент теплообмена является степенной функцией перепада температур между температурой прямого ребра прямоугольного профиля и температурой жидкости, окружающей ребро, даны значения показателя степенной функции, для которых может быть получено решение в замкнутом виде при одномерном распределении температур в ребре. Такое решение в замкнутом виде представлено вместе с характеристиками ребра и его к.п.д. для процесса пузырькового кипения многих часто применяемых жидкостей.